Angle-based Constrained Dominance Principle in MOEA/D for Constrained Multi-objective Optimization Problems

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***Abstract*—This paper proposes a new constraint handling method named Angle-based Constrained Dominance Principle (ACDP). Unlike the original Constrained Dominance Principle (CDP), this approach adopts the angle information of the objective functions to enhance the population's diversity in the infeasible region. To be more specific, given two infeasible solutions, if the angle of the solutions is greater than a given threshold, they are considered to be non-dominated by each other. For a feasible solution and an infeasible solution, if the angle of the solutions is less than a given threshold, the feasible solution is better, otherwise they are non-dominated. To verify the proposed constraint handling approach ACDP, eight test problems CMOP1 to CMOP8 are introduced. The suggested algorithm MOEA/D-ACDP is compared with MOEA/D-CDP and NSGA-II-CDP on CMOP1 to CMOP8. The experimental results demonstrate that ACDP performs better than CDP in the framework of MOEA/D, and MOEA/D-ACDP is significantly better than NSGA-II-CDP, especially on the test instances with the very low ratio of feasible region against the whole objective space.**

***Index Terms*—Constraint-handling Techniques, Constrained Multi-objective Optimization.**

I. Introduction

Constrained Multi-objective Optimization Problems (CMOPs) involve more than one conflicting objectives to be optimized and many constraints to be fulfilled simultaneously. In the real world, most of engineering optimization problems can be formulated as CMOPs [1]. Without loss of generality, a CMOP can be defined as follows [2]:

$$\begin{aligned} \text{minimize} \quad & F(x) = (f_1(x), \ldots, f_m(x))^T \\ \text{subject to} \quad & g_i(x) \geq 0, i = 1, \ldots, q \\ & h_j(x) = 0, j = 1, \ldots, p \\ & x \in \Omega \end{aligned} \tag{1}$$

where $F(x) = (f_1(x), f_2(x), \ldots, f_m(x))^T \in R^m$ is a m-dimensional objective vector, $g_i(x) \geq 0$ defines q inequality constraints, and $h_j(x) = 0$ defines p equality constraints. $x = (x_1, x_2, \ldots, x_n)^T \in R^n$ represents a n-dimensional decision vector. $\Omega = \prod_{i=1}^{n}[a_i, b_i] \subseteq R^n$ denotes the domain of x. A solution x is said to be feasible if it meets $g_i(x) \geq 0, i = 1, \ldots, q$ and $h_j(x) = 0, j = 1, \ldots, p$ at the same time. In CMOPs, there are more than one constraint functions. In order to evaluate the violation of the constraint functions, an overall constraint violation are adopted. The overall constraint violation can be defined as follows:

$$vio(x) = \sum_{i=1}^{q} |min(g_i(x), 0)| + \sum_{j=1}^{p} |h_j(x)| \tag{2}$$

If $vio(x)$ equals to zero, solution x is a feasible solution, otherwise it is an infeasible solution.

Generally, constrained multi-objective evolutionary algorithms (CMOEAs) consist of two parts. One part is to optimize objective functions and the other part is to handle the constraint functions. CMOEAs have been recognized as a promising method to solve CMOPs due to its population-based property, and its ability to achieve an approximation of the Pareto front in a single run.

The existing multi-objective evolutionary algorithms (MOEAs) can be classified into three categories. They are dominance-based, decomposition-based and indicator-based ones. In the dominance-based methods, a solution is selected to enter into next generation according to its non-dominated rank. Typical methods include NSGA-II [3], SPEA-II [4], and PAES-II [5]. In the indicator-based selection category, a solution is selected into next generation according to its contribution to the performance metrics. Representative methods include IBEA [6], R2-IBEA [7] and HypE [8]. In the decomposition-based methods, a solution is selected into the next generation based on the value of aggregation functions.

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Representative methods include IMMOGLS [9], MOEA/D [10] and MOEA/D-DE [11].

Constraint-handling techniques have the capability to balance the objective functions and the constraint functions. The existing constraint-handling mechanisms can be categorized into four groups [12], they are feasibility maintenance, penalty function, separation of constraint violation and objective value, and MOEAs. In the group of feasibility maintenance, special encoding and decoding techniques are adopted to limit the search in the feasible region. In the group of penalty function, the constraints are added to the objectives with predefined or adaptive controlled weights which indicate a preference between the overall constraint violation and objective value. Typical methods of penalty function include segregated penalty functions [13], death penalty functions [14], co-evolutionary penalty functions [15] and adaptive penalty functions [16][17]. Even though the penalty function methods are very popular, the penalty weights are problem-dependent and difficult to tune. Unlike the penalty function approach, the method of separating constraint violation and objective value treats the objective functions and constraint functions separately. The parameters of this method are relatively easy to tune. Typical methods include stochastic ranking (SR) [18], infeasible driven evolutionary algorithm (IDEA) [19], constraint dominate principle (CDP) [20] and epsilon constraint methods [21]. In the group of MOEAs, the constraint-handling technique treats k constraints as k objectives, or the total constraint violation as one objective. Typical methods of this category include COMOGA [22], CW [23] and ATMES [24].

The rest of this paper is organized as follows. Section II introduces the constraint-handling techniques of angle-based constraint-dominance principle. Section III introduces a set of constrained multi-objective optimization problems (CMOPs). Section IV gives the experimental results of MOEA/D-ACDP, MOEA/D-CDP and NSGA-II-CDP, and Section V concludes the paper.

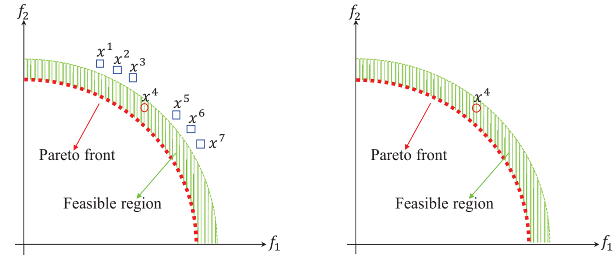
II. Angle-based Constrained Dominance Principle

Constraint dominance principle (CDP) is proposed by Deb [20]. It is a popular and widely used constraint-handling approach. The constraint-handling approach of CDP is defined as follows:

A solution x^i is said to constrained-dominate a solution x^j, if any of the following conditions is true.

1) Solution x^i is feasible and solution x^j is infeasible.
2) Solution x^i and x^j are both infeasible, but solution x^i has a smaller constraint violation than solution x^j.
3) Solution x^i and x^j are both feasible and solution x^i dominates x^j.

This constrained-domination principle has the effect that any feasible solutions is better than any infeasible solutions. However, this constraint-handling technique may neglect many infeasible solutions which have the potential to improve the population's diversity. Fig. 1 shows the effect of the first rule of CDP on the selection procedure in MOEAs.

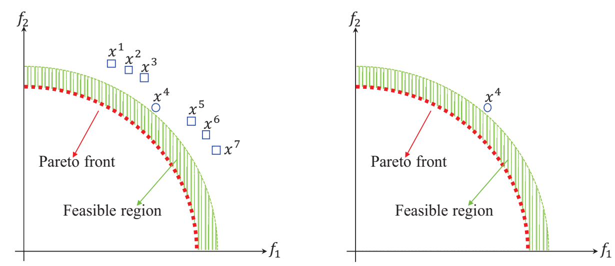


(a) Initial population (b) Evolved population

Fig. 1. The effect of the first rule of CDP, an illustrative example

In Fig. 1(a), solutions in the shadow region are feasible, and the solid cubes are representative points in the Pareto front. There are seven solutions in the objective space. Solution x^4 is feasible and the rest of solutions are infeasible. According to the first rule of CDP, solution x^4 is always better than the other six solutions. After several generations of evolution, six infeasible solutions will be eliminated by solution x^4 as shown in Fig. 1(b).

Fig. 2 shows the effect of the second rule of CDP on the selection procedure in CMOEAs. In Fig. 2(a), all of solutions are infeasible and solution x^4 has the smallest overall constraint violation. The second rule of CDP prefers solution x^4 than other six solutions. Solution x^4 will eliminate all other solutions after several generations of evolution as shown in Fig. 2(b).



(a) Initial population (b) Evolved population

Fig. 2. The effect of the second rule of CDP, an illustrative example

The first two rules of CDP focus on pushing the infeasible solutions into the feasible region. However, it ignores to maintain the population's diversity. If all of the solutions are infeasible, the fitness of the solutions is decided only by the overall constraint violation, which literally converts the CMOP into a single objective optimization problem in the infeasible region. If some solutions are feasible, it starts to enhance the diversity of the population by using the third rule of CDP. However, for the CMOPs with low ratio of feasible solutions in the objective space, it is very difficult to expand the population and increase its diversity of the population.

In order to improve the population's diversity, an angle-based constrained-domination principle (ACDP) is proposed. This constraint-handling method adds an angle constraint to the original CDP. The angle constraint function is defined as follows:

$$c(x^i, x^j) \equiv \arccos \frac{\overline{F}(x^i)^T \overline{F}(x^j)}{||\overline{F}(x^i)||.||\overline{F}(x^j)||} - \theta_{i,j} \leq 0 \qquad (3)$$

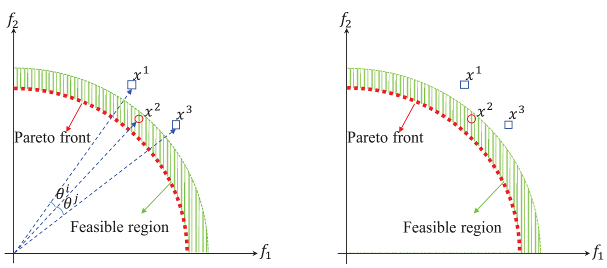
where $\overline{F}(x^i)$ and $\overline{F}(x^j)$ is the normalized objective vector of x^i and x^j respectively, $\theta_{i,j}$ is a parameter defined by users. The $k-th$ individual objective function is normalized as:

$$\overline{f}_k(x) = \frac{f_k(x)}{\sum_{i=1}^{m} f_i(x)} \qquad (4)$$

The constraint-handling approach of ACDP is defined as follows: A solution x^i is said to angle-dominate a solution x^j, if any of the following conditions is true.

1) Solution x^i is feasible and solution x^j is infeasible and they satisfy the constraint function of formula (3) .
2) Solution x^i and x^j are both infeasible and satisfy formula (3), but solution x^i has a smaller constraint violation than solution x^j.
3) Solution x^i and x^j are both feasible and solution x^i dominates x^j.

Fig. 3 shows the effect of the first rule of ACDP, which uses angle information to keep the population's diversity. In Fig. 3(a), the feasible solution x^2 will not dominance solution x^1 or x^3 if the angle θ^i or θ^j is greater than an user-defined parameter θ. The first rule of ACDP enhances the population's diversity as shown in Fig. 3(b).

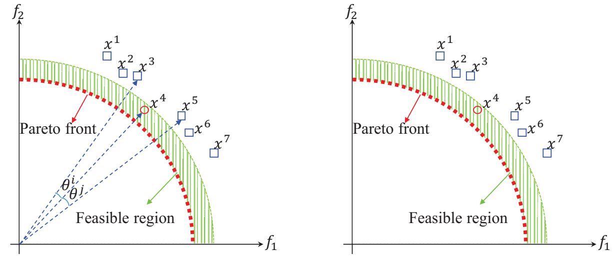


(a) Initial population (b) Evolved population

Fig. 3. The effect of the first rule of ACDP, an illustrative example

Fig. 4 shows the effect of the second rule of ACDP. In Fig. 4(a), the angles of any two peer individuals are greater than an user-defined parameter θ. According to the second rule of ACDP, all of individuals are non-dominated. The second rule of ACDP also helps to increase the population's diversity as shown in Fig. 4(b).

In order to integrate the angle-based constraint dominance principle in the framework of MOEA/D, the third rule of ACDP can be transformed as follows: Solution x^i and x^j are both feasible and $g(x^i|\lambda, z^*)$ is less than $g(x^j|\lambda, z^*)$. $g(x|\lambda, z^*)$ is an aggregation function defined in [10]. The algorithm works as follows.



(a) Initial population (b) Evolved population

Fig. 4. The effect of the second rule of ACDP, an illustrative example

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Algorithm:MOEA/D-ACDP

Input:
1) a CMOP and a stopping criterion.
2) N: the number of subproblems.
3) a set of of N weight vectors: $\lambda^1, \ldots, \lambda^N$.
4) T: the size of the neighborhood.
5) f: the DE parameter.
6) δ: the probability of selecting parents from the neighborhood.
7) n_r: the maximal number of solutions replaced by a child.
8) θ: the parameter of ACDP.

Output: A set of non-dominated feasible solutions NS;

Step 1: Initialization:
a) Decompose the CMOP into N subproblems associated with $\lambda^1, \ldots, \lambda^N$.
b) Generate an initial population $P = \{x^1, \ldots, x^N\}$.
c) Initialize $z^* = (z_1, \ldots, z_m)$ by setting $z_j = \min_{1 \leq i \leq N} f_j(x^i)$ and set $NS = P$.
e) Compute the Euclidean distance between any two weight vectors and obtain T closest weight vectors to each weight vector. For each $i = 1, \ldots, N$, set $B(i) = \{i_1, \ldots, i_T\}$, where $\lambda^{i_1}, \ldots, \lambda^{i_T}$ are the T closest weight vectors to λ^i.

Step 2: Population update
For $i = 1, \ldots, N$, do
a) Generate a random number r from $[0, 1]$. If $r < \delta, S = B(i)$, else $S = \{1, \ldots, N\}$
b) Set $r_1 = i$ and randomly select two indexes from S, and generate $\mathbf{y}^i = \mathbf{x}^{r_1} + f * (\mathbf{x}^{r_2} - \mathbf{x}^{r_3})$.
c) Perform a mutation operator on $\mathbf{y}^i$, and repair $\mathbf{y}^i$.
d) Update z^*, For each $j = 1, \ldots, m$, if $z_j^* > f_j(\mathbf{y}^i)$, then set $z_j^* = f_j(\mathbf{y}^i)$.
e) **Update of Solutions**: Set $c = 0$ and then do the following:
   1) If $c = n_r$ or S is empty, go to **Step3**. Ohterwise, select an index j from S randomly.
   2.1) If $vio(y^i) == vio(x^j)$ and $g(\mathbf{y}^i|\lambda^j, z^*) \leq g(\mathbf{x}^j|\lambda^j, z^*)$, then set $\mathbf{x}^j = \mathbf{y}^i$ and $c = c+1$.
   2.2) If $vio(y^i) < vio(x^j)$ and $c(y^i, x^j) \leq 0$, then set $\mathbf{x}^j = \mathbf{y}^i$ and $c = c+1$.
   3) Remove j from S and go to 1).
f) Set $NS = P$.

Step 3: Termination If stopping criteria are satisfied, output NS. Otherwise, go to **Step 2**.

III. Test Problems

Most of the existing CMOPs (e.g. CTP [25] and CF [26]) have high ratio of feasible solutions (RFS) in the objective space. These test instances can not verify the effectiveness of the proposed constraint-handling technique ACDP. Because ACDP is specifically designed to solve CMOPs with low RFS. In order to verify the constraint-handling method ACDP, we select eight test instances from our earlier work [27]. The detail formulas are listed in Table I.

In the eight test problems, four test instances CMOP3-CMOP6 have a low ratio of feasible solutions in the objective space, and the rest of test problems CMOP1-CMOP2 and CMOP7-CMOP8 have a high ratio of feasible solutions in the objective space. The aim of the proposed constraint-handling technique ACDP is to enhance the population's diversity in the infeasible area. Test instances CMOP3-CMOP6 will help to verify the effectiveness of the proposed ACDP. Test instances CMOP1-2 and CMOP7-8 will help to verify the superiority of ACDP against CDP.

IV. EXPERIMENTAL STUDY

A. Experimental Settings

To verify the effectiveness of ACDP, two popular algorithms (i.e., MOEA/D-CDP and NSGA-II-CDP) are adopted in our experiments. The parameter settings are defined as follows:

1) Setting for reproduction operators: The mutation probability $Pm = 1/n$ (n is the number of decision variables) and its distribution index is set to be 20. For the DE operator, we set $CR = 1.0$ and $f = 0.5$.
2) Population size: $N = 300$.
3) Number of runs and stopping condition: Each algorithm runs 30 times independently on each test problems. The algorithm stops until 300 000 function evaluations.
4) Neighborhood size: $T = 20$.
5) Probability used to select in the neighbourhood: $\delta = 0.9$.
6) The maximal number of solutions replaced by a child: $n_r = 2$.
7) The angle constraint parameter: $\theta = \frac{T}{2N}0.5\pi$

B. Performance Metric

To compare the performance of MOEA/D-CDP, MOEA/D-ACDP and NSGA-II-CDP, two popular metrics - inverted generation distance (IGD)[28] and relative hypervolume indicator (I_H^-) [29] are adopted. The definitions of IGD and I_H^- are given as follows:

- **Inverted Generational Distance** (IGD):

$$\begin{cases} IGD(P^*, A) = \frac{\sum_{y^* \in P^*} d(y^*, A)}{\|P^*\|} \\ d(y^*, A) = \min_{y \in A}\{\sqrt{\sum_{i=1}^{m}(y_i^* - y_i)^2}\} \end{cases} \tag{5}$$

where P^* is the set of representative points in the true Pareto front, A is an approximate Pareto front set achieved by algorithms. IGD metric denotes the distance between P^* and A, the smaller value of IGD indicates the better performance of both convergence and diversity.

- **Relative Hypervolume Indicator** (I_H^-):

$$\begin{cases} I_H^-(A, P^*, R) = I_H(P^*, R) - I_H(A, R) \\ I_H(P^*, R) = Vol_{v \in P^*}(v) \\ I_H(A, R) = Vol_{v \in A}(v) \end{cases} \tag{6}$$

where $I_H(P^*, R)$ is defined as the volume enclosed by P^* and the reference vector $R = (R_1, \ldots, R_m)$. $I_H(A, R)$ is defined as the volume enclosed by A and the reference vector R. I_H^- simultaneously considers the distribution of the obtained Pareto front A and its vicinity to the true Pareto front. For CMOP1-2 and CMOP7-8, the reference point R is set to $(1.2, 1.2)^T$. For CMOP3-CMOP6, R is set to $(1.6, 1.6)^T$. The smaller value of I_H^- represents the better performance of both diversity and convergence.

C. Experimental Results and Discussions

Table II and Table III show the mean values of IGD and I_H^- calculated by MOEA/D-CDP and MOEA/D-ACDP. The Wilcoxon's rank sum test values of IGD and I_H^- are set at 0.05 significant level. For test instances CMOP3-CMOP6, MOEA/D-ACDP is significantly better than MOEA/D-CDP. For the rest of test problems CMOP1-2 and CMOP7-8, MOEA/D-CDP and MOEA/D-ACDP have no significant difference on these two metrics. Fig. 5 shows the populations with the best IGD metrics among 30 independent running using MOEA/D-CDP and MOEA/D-ACDP. It can be observed that MOEA/D-ACDP has the better approximation of Pareto front than MOEA/D-CDP on test instances CMOP3,CMOP5 and CMOP6. For the rest of test problems, MOEA/D-CDP and MOEA/D-ACDP have similar approximation of Pareto front. The above experimental results verify that the proposed constraint-handling method ACDP is very effective in the framework of MOEA/D. It significantly increases the performance of population's diversity, especially for the CMOPs which have low ratio of feasible solutions.

Table IV and Table V show the statistic results of IGD and I_H^- metrics achieved by NSGA-II-CDP and MOEA/D-ACDP. The Wilcoxon's rank sum test values of IGD and I_H^- are also set at 0.05 significant level. It can be observed that MOEA/D-ACDP is significantly better than NSGA-II-CDP on most of test instances in terms of IGD metric except CMOP7. On CMOP7, the mean value of IGD of MOEA/D-ACDP is still better than that of NSGA-II-CDP. In terms of I_H^- metric, MOEA/D-ACDP is significantly better than NSGA-II-CDP on all of test instances. Fig. 6 shows the populations with the best IGD metrics among 30 independent running using NSGA-II-CDP. It is clear that MOEA/D-ACDP has the better approximation of Pareto front than NSGA-II-CDP on all of test instances.

Fig. 7 shows the box plots of IGD and I_H^- values in 30 independent running by using MOEA/D-ACDP, MOEA/D-CDP and NSGA-II-CDP. It is clear that MOEA/D-ACDP has the smaller mean values of IGD on most of test problems than NSGA-II-CDP except CMOP7, and it has the smaller mean values of I_H^- on all of test problems than NSGA-II-CDP. MOEA/D-ACDP has the smaller mean IGD and I_H^- values

TABLE I
CMOP1-CMOP8 TEST PROBLEMS

Name	Objective Functions	Constraint Functions and Parameters	Geometry of PF	RFS
CMOP1	$f_1(x) = x_1 + g_1(x)$ $f_2(x) = 1 - x_1^2 + g_2(x)$ $g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2$ $g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2$	$c(x) = sin(a\pi x_1) - 0.5 \geq 0$ $J_1 = \{j \mid j$ is odd and $2 \leq j \leq n\}$ $J_2 = \{j \mid j$ is even and $2 \leq j \leq n\}$ $a = 20, n = 30, x_j \in [0, 1]$	disconnect and concave	high
CMOP2	$f_1(x) = x_1 + g_1(x)$ $f_2(x) = 1 - \sqrt{x_1} + g_2(x)$ $g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2$ $g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2$	$c(x) = sin(a\pi x_1) - 0.5 \geq 0$ $J_1 = \{j \mid j$ is odd and $2 \leq j \leq n\}$ $J_2 = \{j \mid j$ is even and $2 \leq j \leq n\}$ $a = 20, n = 30, x_j \in [0, 1]$	disconnect and convex	high
CMOP3	$f_1(x) = x_1 + g_1(x)$ $f_2(x) = 1 - x_1^2 + g_2(x)$ $g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2$ $g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2$	$c_1(x) = (a - g_1(x)) * (g_1(x) - b) \geq 0$ $c_2(x) = (a - g_2(x)) * (g_2(x) - b) \geq 0$ $J_1 = \{j \mid j$ is odd and $2 \leq j \leq n\}$ $J_2 = \{j \mid j$ is even and $2 \leq j \leq n\}$ $a = 0.51, b = 0.5, n = 30, x_j \in [0, 1]$	concave	low
CMOP4	$f_1(x) = x_1 + g_1(x)$ $f_2(x) = 1 - \sqrt{x_1} + g_2(x)$ $g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2$ $g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2$	$c_1(x) = (a - g_1(x)) * (g_1(x) - b) \geq 0$ $c_2(x) = (a - g_2(x)) * (g_2(x) - b) \geq 0$ $J_1 = \{j \mid j$ is odd and $2 \leq j \leq n\}$ $J_2 = \{j \mid j$ is even and $2 \leq j \leq n\}$ $a = 0.51, b = 0.5, n = 30, x_j \in [0, 1]$	convex	low
CMOP5	$f_1(x) = x_1 + g_1(x)$ $f_2(x) = 1 - x_1^2 + g_2(x)$ $g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2$ $g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2$	$c_1(x) = (a - g_1(x)) * (g_1(x) - b) \geq 0$ $c_2(x) = (a - g_2(x)) * (g_2(x) - b) \geq 0$ $c_3(x) = sin(c\pi x_1) - 0.5 \geq 0$ $J_1 = \{j \mid j$ is odd and $2 \leq j \leq n\}$ $J_2 = \{j \mid j$ is even and $2 \leq j \leq n\}$ $a = 0.51, b = 0.5, c = 20, n = 30, x_j \in [0, 1]$	disconnect and concave	low
CMOP6	$f_1(x) = x_1 + g_1(x)$ $f_2(x) = 1 - \sqrt{x_1} + g_2(x)$ $g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2$ $g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2$	$c_1(x) = (a - g_1(x)) * (g_1(x) - b) \geq 0$ $c_2(x) = (a - g_2(x)) * (g_2(x) - b) \geq 0$ $c_3(x) = sin(c\pi x_1) - 0.5 \geq 0$ $J_1 = \{j \mid j$ is odd and $2 \leq j \leq n\}$ $J_2 = \{j \mid j$ is even and $2 \leq j \leq n\}$ $a = 0.51, b = 0.5, c = 20, n = 30, x_j \in [0, 1]$	disconnect and convex	low
CMOP7	$f_1(x) = x_1 + g_1(x)$ $f_2(x) = 1 - x_1^2 + g_2(x)$ $g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2$ $g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2$	$c_k(x) = ((f_1 - p_k)cos\theta - (f_2 - q_k)sin\theta)^2/a^2$ $+((f_1 - p_k)sin\theta + (f_2 - q_k)cos\theta)^2/b^2 \geq 1$ $c_{10}(x) = sin(c\pi x_1) - 0.5 \geq 0$ $p = [0, 1, 0, 1, 2, 0, 1, 2, 3]$ $q = [1.5, 0.5, 2.5, 1.5, 0.5, 3.5, 2.5, 1.5, 0.5]$ $J_1 = \{j \mid j$ is odd and $2 \leq j \leq n\}$ $J_2 = \{j \mid j$ is even and $2 \leq j \leq n\}$ $a^2 = 0.1, b^2 = 0.4, \theta = -0.25\pi$ $c = 20, n = 30, x_j \in [0, 1], k = 1, \ldots, 9$	disconnect and concave	high
CMOP8	$f_1(x) = x_1 + g_1(x)$ $f_2(x) = 1 - \sqrt{x_1} + g_2(x)$ $g_1(x) = \sum_{j \in J_1} (x_j - sin(0.5\pi x_1))^2$ $g_2(x) = \sum_{j \in J_2} (x_j - cos(0.5\pi x_1))^2$	$c_k(x) = ((f_1 - p_k)cos\theta - (f_2 - q_k)sin\theta)^2/a^2$ $+((f_1 - p_k)sin\theta + (f_2 - q_k)cos\theta)^2/b^2 \geq 1$ $c_{10}(x) = sin(c\pi x_1) - 0.5 \geq 0$ $p = [0, 1, 0, 1, 2, 0, 1, 2, 3]$ $q = [1.5, 0.5, 2.5, 1.5, 0.5, 3.5, 2.5, 1.5, 0.5]$ $J_1 = \{j \mid j$ is odd and $2 \leq j \leq n\}$ $J_2 = \{j \mid j$ is even and $2 \leq j \leq n\}$ $a^2 = 0.1, b^2 = 0.4, \theta = -0.25\pi$ $c = 20, n = 30, x_j \in [0, 1], k = 1, \ldots, 9$	disconnect and convex	high

on test problems CMOP3-6 than MOEA/D-CDP. Thus, it can be concluded that MOEA/D-ACDP performs better than other two popular algorithms MOEA/D-CDP and NSGA-II-CDP.

V. Conclusion

The paper proposed a new constraint-handling technique named ACDP. It utilizes the angle information of any two peer individuals to enhance the diversity of the population. To verify the proposed ACDP, we selected eight test problems and compared the proposed algorithm MOEA/D-ACDP with MOEA/D-CDP and NSGA-II-CDP. Experimental results show that the proposed method is very effective to enhance the population's diversity, especially on CMOP3-CMOP6 which have low ratio of feasible solutions. MOEA/D-ACDP is significantly better than MOEA/D-CDP and NSGA-II-CDP on most cases. The future work includes studying the parameter setting of θ introduced by ACDP and solving several real engineering optimization problems to further demonstrate the effectiveness of ACDP.

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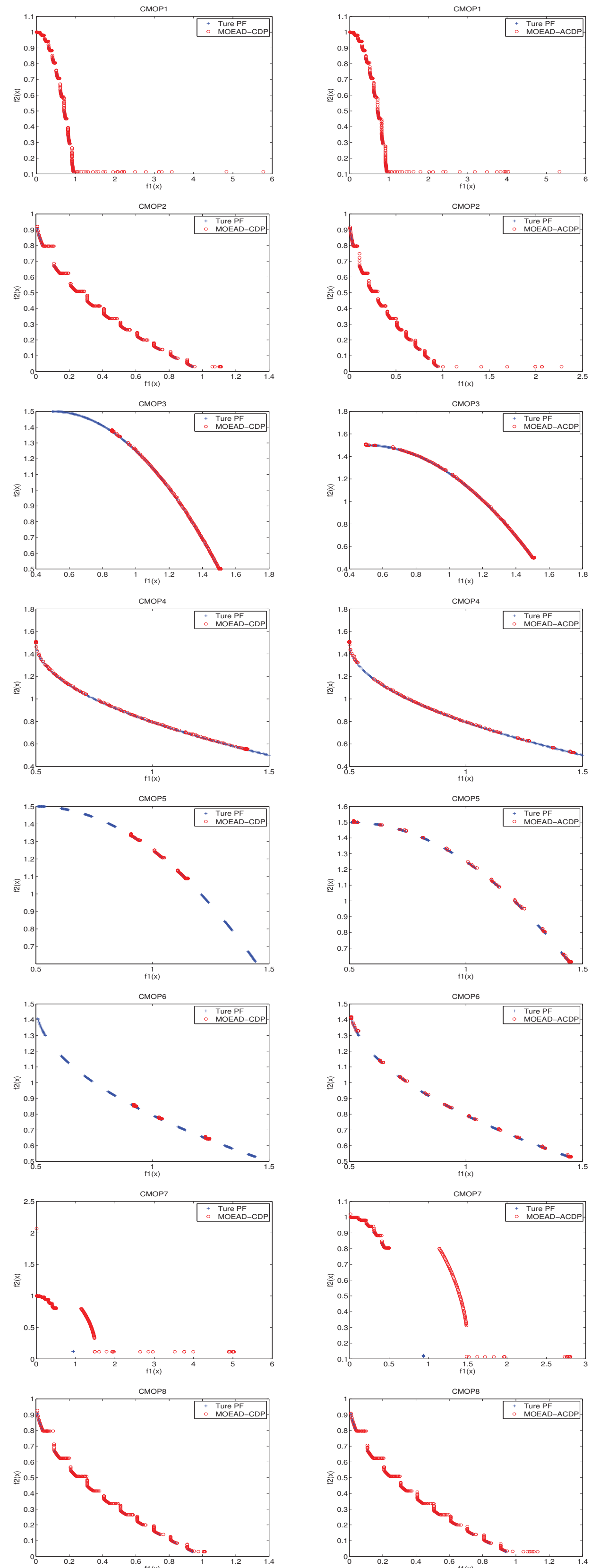

Fig. 5. The final populations with the best IGD metric in 30 independent runs by using MOEA/D-CDP and MOEA/D-ACDP

TABLE II
IGD VALUES OF MOEA/D-CDP AND MOEA/D-ACDP.

Instance	MOEA/D-CDP		MOEA/D-ACDP		Wilcoxon's Rank	
–	Mean	Std.	Mean	Std.	p-value	h-value
CMOP1	3.50E-03	2.95E-04	3.55E-03	2.77E-04	7.19E-01	0.00E+00
CMOP2	3.14E-03	2.81E-04	3.21E-03	3.74E-04	4.53E-01	0.00E+00
CMOP3	2.54E-01	4.61E-02	**3.81E-02**	2.97E-02	1.73E-06	**1.00E+00**
CMOP4	2.15E-01	5.02E-02	**4.29E-02**	2.50E-02	1.92E-06	**1.00E+00**
CMOP5	3.00E-01	3.52E-02	**4.02E-02**	3.05E-02	1.73E-06	**1.00E+00**
CMOP6	2.77E-01	3.00E-02	**3.88E-02**	2.52E-02	1.73E-06	**1.00E+00**
CMOP7	1.55E-01	5.22E-02	1.58E-01	5.29E-02	4.78E-01	0.00E+00
CMOP8	6.98E-03	8.03E-03	7.54E-03	1.55E-02	7.19E-02	0.00E+00

TABLE III
I_H^- VALUES OF MOEA/D-CDP AND MOEA/D-ACDP.

Instance	MOEA/D-CDP		MOEA/D-ACDP		Wilcoxon's Rank	
–	Mean	Std.	Mean	Std.	p-value	h-value
CMOP1	2.26E-03	2.32E-04	2.30E-03	2.28E-04	7.50E-01	0.00E+00
CMOP2	2.30E-03	4.52E-04	2.36E-03	5.46E-04	6.58E-01	0.00E+00
CMOP3	2.51E-01	4.18E-02	**4.86E-02**	3.60E-02	1.73E-06	**1.00E+00**
CMOP4	2.86E-01	6.36E-02	**5.36E-02**	2.90E-02	1.73E-06	**1.00E+00**
CMOP5	2.64E-01	2.14E-02	**4.48E-02**	2.97E-02	1.73E-06	**1.00E+00**
CMOP6	3.32E-01	3.35E-02	**5.29E-02**	3.17E-02	1.73E-06	**1.00E+00**
CMOP7	2.58E-01	2.67E-02	2.60E-01	2.62E-02	7.19E-01	0.00E+00
CMOP8	9.33E-03	1.52E-02	1.14E-02	3.52E-02	7.86E-02	0.00E+00

TABLE IV
IGD VALUES OF NSGA-II-CDP AND MOEA/D-ACDP.

Instance	NSGA-II-CDP		MOEA/D-ACDP		Wilcoxon's Rank	
–	Mean	Std.	Mean	Std.	p-value	h-value
CMOP1	1.58E-01	1.41E-01	**3.55E-03**	2.77E-04	1.73E-06	**1.00E+00**
CMOP2	2.08E-01	5.60E-02	**3.21E-03**	3.74E-04	1.73E-06	**1.00E+00**
CMOP3	3.56E-01	1.02E-01	**3.81E-02**	2.97E-02	1.73E-06	**1.00E+00**
CMOP4	3.00E-01	7.11E-02	**4.29E-02**	2.50E-02	1.73E-06	**1.00E+00**
CMOP5	3.84E-01	1.09E-01	**4.02E-02**	3.05E-02	1.73E-06	**1.00E+00**
CMOP6	3.25E-01	5.83E-02	**3.88E-02**	2.52E-02	1.73E-06	**1.00E+00**
CMOP7	2.09E-01	2.97E-01	**1.58E-01**	5.29E-02	1.59E-01	0.00E+00
CMOP8	5.94E-01	3.88E-01	**7.54E-03**	1.55E-02	1.73E-06	**1.00E+00**

TABLE V
I_H^- VALUES OF NSGA-II-CDP AND MOEA/D-ACDP.

Instance	NSGA-II-CDP		MOEA/D-ACDP		Wilcoxon's Rank	
–	Mean	Std.	Mean	Std.	p-value	h-value
CMOP1	2.58E-01	1.09E-01	**2.30E-03**	2.28E-04	1.73E-06	**1.00E+00**
CMOP2	3.19E-01	6.38E-02	**2.36E-03**	5.46E-04	1.73E-06	**1.00E+00**
CMOP3	3.12E-01	4.45E-02	**4.86E-02**	3.60E-02	1.73E-06	**1.00E+00**
CMOP4	3.74E-01	7.59E-02	**5.36E-02**	2.90E-02	1.73E-06	**1.00E+00**
CMOP5	3.02E-01	3.42E-02	**4.48E-02**	2.97E-02	1.73E-06	**1.00E+00**
CMOP6	3.81E-01	7.90E-02	**5.29E-02**	3.17E-02	1.73E-06	**1.00E+00**
CMOP7	3.45E-01	6.83E-02	**2.60E-01**	2.62E-02	1.73E-06	**1.00E+00**
CMOP8	6.62E-01	2.42E-01	**1.14E-02**	3.52E-02	1.73E-06	**1.00E+00**

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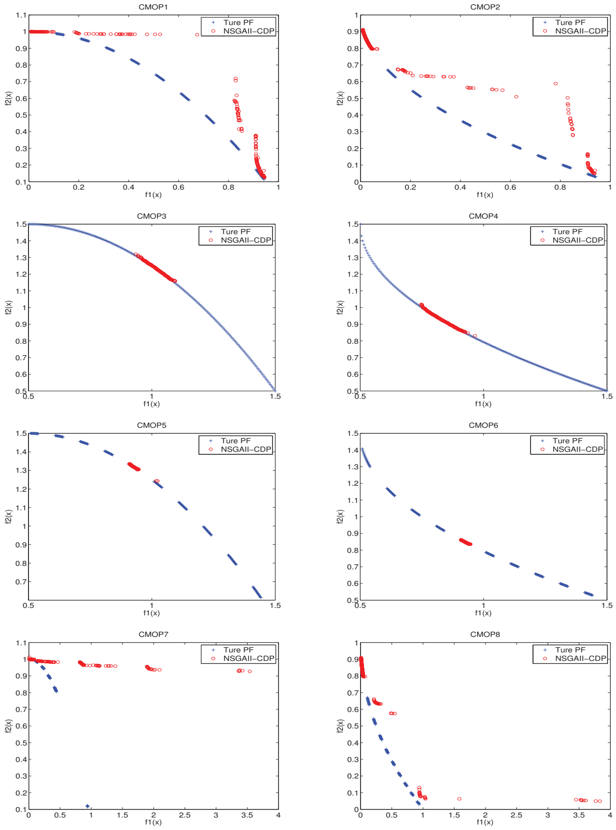

Fig. 6. The final populations with the best IGD value in 30 independent runs by using MOEA/D-CDP and MOEA/D-ACDP

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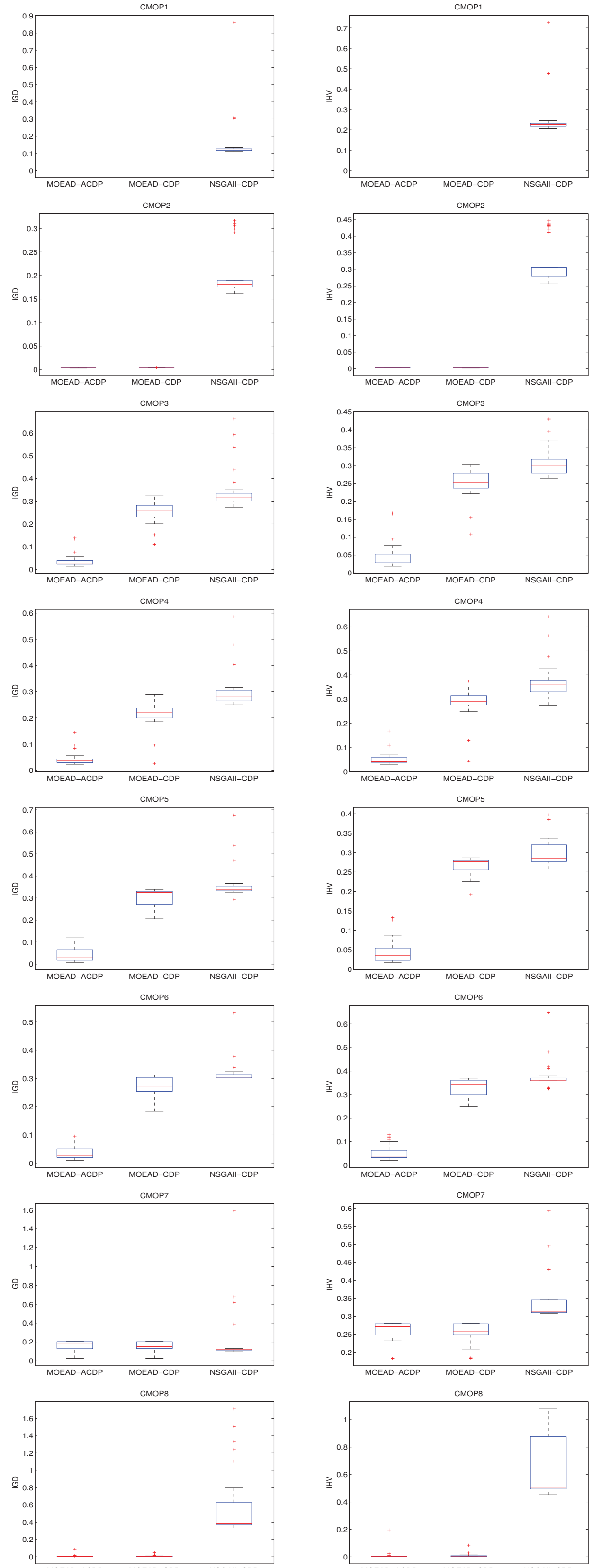

Fig. 7. The boxplot of IGD and I_H^- metrics of MOEA/D-ACDP, MOEA/D-CDP and NSGA-II-CDP